Portfolio Optimization Using Mean-Semi Variance approach with Artificial Neural Networks: Empirical Evidence from Pakistan

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ARTICLE

**ABSTRACT**

**Purpose:** The challenge of managing a portfolio effectively is allocating capital among numerous stock holdings to achieve maximum profit. Therefore, the purpose of this study is to guide investors in developing optimal portfolios in the stock market of Pakistan.

**Design/Methodology/Approach:** To pick and optimize a portfolio in the most effective way possible, we used the daily closing stock prices of a sample of listed firms at the Pakistan stock exchange. The study applied the mean semi-variance approach and compared the performance of portfolios with equally weighted portfolios under artificial neural networks and historical-based return estimation in Pakistan.

**Findings:** The result shows that artificial neural network-based estimation of the expected return vector has outperformed the historical return estimation under mean semi-variance portfolio optimization and constrained mean semi-variance portfolios based on the Sharp ratio in Pakistan.

**Implications/Originality/Value:** The study suggests that investors, fund managers, and portfolio analysts should focus on the more sophisticated neural network-based choice for the development of portfolios in the equity market of Pakistan.

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**Introduction**

An investor's goal should be to increase their portfolio profit while minimizing risk exposure. A
portfolio can reduce risk by taking advantage of excess returns compared to investing in individual stocks. The challenge of managing a portfolio effectively is allocating capital among numerous stock holdings to achieve maximum profit. The investor's profit goal, the characteristics of the stock market, and the length of time they want to invest are some factors that influence the investment strategy they choose. The fundamental concept behind a portfolio may be traced back to Markowitz (1952) and his simple mean-variance technique. The mean-variance framework is a parametric model that can streamline the analysis of a single time frame (Markowitz, 1952; Chan, 1999; Husnain, Hassan, & Lamarque, 2016b). When developing investment plans, it is necessary to make informed decisions regarding selecting stocks in which to invest over a predetermined amount of time. This option is influenced by several elements, including the aim of the investment, the features of market factors, and the desired amount of time spent investing. The mean-variance framework (M-V) that Markowitz developed serves as the basis for portfolio investment decisions. The primary goal of investors is to minimize risk. Thus, this framework was developed to accomplish this objective. Many people believe that the M-V model developed by Markowitz is the optimal option for investments within a single time frame.

The term "neural system" refers to a type of computing system comprised of many organised devices and has a rapid response in real time to inputs from the outside world (Caudil, 1989; Odom, 1990; Caudil, 1992). In 1943, the inspective methods of ANNs were first developed to develop rules and regulations for credible and relevant apparatuses. Therefore, the phenomenon of artificial intelligence is employed to compete with human intelligence to find solutions to investment difficulties (Zahedi, 1991). This strategy appears to have an advantage over the usual Mean-Variance framework because the latter cannot explain the influence of incorrect calculations regarding portfolio selections (Jorion, 1992). In the most recent study, the researchers used predictors from neural networks to consider the impact of errors (Ceria & Stubbs, 2016; Braun, 2017).

Numerous applications, such as the incorporation of the M-V framework into the creation of contemporary portfolio theory, have expressed their satisfaction with the proficient portfolio choice as a highly crucial option (Markowitz, 1952; Elton, 1976). The prudent investors constantly look at risks and return together rather than addressing them in isolation from one another (Abdulnasser, 2015).

The challenge of managing a portfolio effectively is allocating capital among numerous stock holdings to achieve maximum profit (Husnain, Hassan, & Lamarque, 2016a). Therefore, the purpose of this study is to guide investors in developing optimal portfolios in the stock market of Pakistan. To pick and optimize a portfolio in the most effective way possible, we used the daily closing stock prices of a sample of listed firms at the Pakistan stock exchange. The study applied the mean semi-variance approach and compared the performance of portfolios with equally weighted portfolios under artificial neural networks and historical-based return estimation in Pakistan. In addition, we applied additional restrictions for portfolio optimization of Pakistan stock exchange-listed businesses to lessen the dependence on only one metric and increase overall efficiency. This was done to improve overall efficiency. Therefore, various methods are utilized in continuous optimization (Fernández, 2007; DeMiguel, 2009; Kritzman, 2010; Coqueret, 2015; Hatemi, 2015). The result shows that artificial neural network-based estimation of the expected return vector has outperformed the historical return estimation under mean semi-variance portfolio optimization and constrained mean semi-variance portfolios based on the Sharp ratio in Pakistan. The study suggests that investors, fund managers, and portfolio analysts...
should focus on the more sophisticated neural network-based choice for developing portfolios in the equity market of Pakistan.

**Research Framework and Hypothesis Development**

A succinct charting is performed here in subsequent stature, revealing the main actions that will adhere to this research, beginning after the fundamental price records cultivating the entire portfolio optimisation.

![Figure 1: Research Design used for Portfolio Optimization (Author’s Construct)](image)

For portfolio improvement, two tricks are regularly utilized; the first is the application of the different requirements (Iqbal et al., 2019), and the second is lowering the extraordinary qualities in the boundaries of information sources (Ledoit, 2004). Given supporting writing for ANNs and the mean semi variance markovian model (Fernandez, 2007; Estrada, 2008; Kolm, 2014; Hatemi, 2015; Iqbal, 2019), we in this area will propose speculation for Portfolio Optimization in Pakistan. Various investigations have additionally grasped that the expansion in the portfolio comes back with different imperatives used for portfolio enhancement (Bessler, 2017). Numerous analysts used higher and lowered adaptable weightages to upgrade the Sharpe ratios, and eventually, the group returns expanded. Thus, we will utilize various systems to get flexible loads and reasonable limitations to discover the ideal portfolio returns (DeMiguel, 2009; Kritzman, 2010; Coqueret, 2015). Right off the bat, we will apply portfolio constraints of equivalent weights called (1/n) naïve portfolio. Consequently, the first hypothesis for this study comes out to be

**Hypothesis 1:** “Portfolio based on historical return estimation outperformed the ANNs predicated returns under naïve diversification strategy”
The portfolio distribution is a preference for allocation amongst riskless and critical groups of assets (Tobin, 1958). A spending imperative explicitly upgrades profits by making equilibrium amongst expenditure and portfolio prerequisites (Dickinson, 2001). A few researchers likewise indicated that spending imperatives improve returns by bringing down the variances of any stock in this manner, making important riches increment and decidedly complementing the portfolio revenues (Chen, 2018, Iqbal et al., 2019). In this manner, we will apply budget constraint limitation in our advancement of critical thinking and develops a premise on previously stated assumptions,

**Hypothesis 2:** “Portfolio based on historical return estimation outperformed the ANNs predicated returns under mean semi-variance optimization strategy”

An examination analyzed the highlights of target-risk techniques identified with a file hypothesis. They broke down in the situation of normal alternative approaches like least variation and naïve portfolios (Iqbal et al., 2019). These marketplaces come up intermittently with high-level predictable returns and immense flimsiness (Harvey, 1995; Chen, 2018). Consequently, expansion in advancing marketplace stocks in any portfolio expressively lessens its fragility and rises projected returns through huge risk, and brings ranges back (Bodie, 2013; Chen, 2018, Iqbal et al, 2019).

**Hypothesis 3:** “Portfolio based on historical return estimation outperformed the ANNs predicated returns under mean semi-variance optimization strategy by setting budget constraints”

On the way to quantifying exchange costs, various developers give various perspectives. They may upset numerous creation capacities (Coase, 1992; 1998) for limiting this cost, and many risky castigations may emerge accordingly. We represented these charges in portfolio improvement as in our sight these charges may cost disproportionately higher whenever overlooked as shown by numerous analysts (Michael, 2008; Glen, 2011; Deng, 2017; Iqbal et al., 2019). Subsequently, we developed the following hypothesis,

**Hypothesis 4:** “Portfolio based on historical return estimation outperformed the ANNs predicated returns under mean semi-variance optimization strategy by setting transaction cost constraints”

**Data and Research Methodology**
This section discusses the data used in the study and the research methodology used to develop the optimal portfolios.

**Data Description**
To pick and optimize a portfolio in the most effective way possible, we made use of the closing stock prices of fifty listed firms in Pakistan. For this study, the time-period study started in January 2017 and ended in December 2021. We utilized the KIBOR rates as our risk-free interest rate. To proceed with the study in a more thorough manner, we will utilize the KSE -100 index as a benchmark. We relied on market prices since they take into account all relevant information and are calculated in a way that eliminates the risk of investors seeing abnormally high profits. The data has been collected from the website of the Pakistan stock exchange i.e. the data portal of the Pakistan stock exchange.

**Research Methodology**
The study employed the following research methodology in the present study.
Artificial Neural Networks-ANNs
In the late 1940s, Hebb portrayed the primary lead of neuronal learning. Along these lines, Hebbian coordinating of presynaptic and postsynaptic activity can liberally alter the dynamic characteristics of the synoptical affiliation and, therefore, empower or control signal broadcasts (Hebb, 1949; 2005). Neural frameworks are produced using direct mechanical assemblies working in a closely resembling manner. These mechanical assemblies are driven by characteristic tactile courses of action. Normally, framework measurements are corrected by the relationship among devices. Several can figure out the neuronal framework to work away a height by fluctuating the connections (loads) calculations amongst mechanical assemblies. Mostly, neural frameworks are composed and sorted out; data triggers the objective return. The commitments to a neuronal unit join its inclination and the total of its subjective information resources (using the internal thing). The return of a neuronal unit depends upon the neuron's springs of information and its trade work. The finest composition to use hangs on such a matter to be tackled by the structure, as shown in Picture 2 by Rosenblatt (1962).

Figure 2: Neural Network Functioning

Method of Feedforward Backpropagation
ANNs use feedforward backpropagation calculations incorporating the following equation,
\[ I_i = \sum_j w_{ij} O_j + \phi_i \]
\[ O_i = \frac{1}{1+e^{\phi_i}} \]
Amongst feedforward net there lies three kinds of handling segments 1) input, 2) output, and 3) hidden. The first segment gets signals from elsewhere and lies in the substandard most layers of ANNs, concealed parts do not intrude with outside henceforth, they are covered up. Output segments impart signs out there so rest in the extremely elevated layer. Associations amongst layers are not permitted straightforwardly yet finished with interfacing vectors 'W', which choose what sources of info are to be handled and what discretionary data is to be given (Williams, 1986; Tam, 1992). Backpropagation calculation is extremely powerful as it can appoint loads to multilayers at once (Rosenblatt, 1962). If we have 's' several models with \( X_i = x_{i1}, x_{i2}, \ldots, x_{im} \)
input vector, and output vector as \( D_i = d_{i1}, d_{i2}, ..., d_{in} \). In concurrence with forwards dissemination \( X_i \) is provided an input degree and productivity is generated as \( Y_i = y_{i1}, y_{i2}, ..., y_{in} \) established on weights. The significance of \( Y_i \) is contrasted by \( D_i \) all together through the squaring of error terms as \((y_{ij} - d_{ij})^2\) at each output element. Consequently, the error function is calculated as:

\[
E = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{(y_{ij} - d_{ij})^2}{2}
\]  (2)

Following the calculation of errors for lowering errors, the model aims at minimizing the disparities between yield delivered and actual yield vectors. By adjusting weightages (in eq no. 3), \( \varepsilon \) is characterized as the rate of convergence (Rosenblatt, 1962; Williams, 1986; Lippmann, 1987)

\[
\Delta w_{ij} = - \frac{\partial E}{\partial w_{ij}} \varepsilon ,
\]  (3)

The error term calculated in Eq. 2 is back transmitted in the second step from yield to response element. Weights are portrayed by proliferation at every single point, therefore, \( \frac{\partial E}{\partial w_{ij}} \) stays calculated on each level by employing the rule of the sequence revealed in Eq. 4

\[
\frac{\partial E}{\partial w_{ij}} = \frac{\partial E}{\partial O_i} \frac{\partial O_i}{\partial l_i} \frac{\partial l_i}{\partial w_{ij}}
\]  (4)

**Predicting Time Series Using ANNs**

The gauge of future estimations of money-related components is a basic part of monetary models yet in addition to some business decisions. In this study, another quantifiable methodology is introduced, which attempts to vanquish the issue of such nonlinear impacts (Azoff, 1994; Kaastra, 1996). Our examination keeps up a spot with the logical field of time arrangement gauging, a crucial subfield of econometrics. The purpose of time game plan examination is to focus information on a given data course of action, including recognitions after some time. This information is used to create a model of the stream, called get ready, which chooses the data course of action. Such a model can be used to gauge future estimations of the time course of action. For distinguishing proof of the method, straight models like direct autoregressive strategies (AR) and autoregressive moving midpoints are standard mechanical assemblies of econometrics at any rate (Commandeur & Koopman, 2007).

Anyway, accurate familiarity exhibits immediate simulations that are not commonly the safest way to pact with perceiving a strategy and not for the most part pass on the best estimate results. Granger, (1993) discusses concealed non-linearity, which necessitates the allotment of non-linear structures. Particularly during monetary emergencies non-linearities may be found. From the start of the 2000s, an extensive proportion of non-linear systems has risen. (Granger and Teräsvirta, 1993). In this manner, we utilized a parametric strategy for estimation as we used a couple of quantities of fixed factors.

**The Model for Autoregression**

Time series guesstimate is the sub-element of econometrics subject and stands with cracking down the fundamental drift of an array of sequentially witnessed prior period data, identified such as time series. ‘T’ time series is an agreement of any discerned value of a random parameter as \( t = 1, 2, ..., n \). On the off chance that simply the time course of action is given, we have to recognize the method which chooses the time game plan using only the information given by the game plan. Thusly, the strategy is separated into an area that we can choose or envision and an unpredictable part. To make a supportive model of a methodology, be that as it may, much as
could be required should be explained by the underlying fragment. On the off chance that we have 'n' past qualities, at that point on relapsing x T esteems is named as Auto Regression.

\[ Y_t = f(Y_{t-1}) + e' \]  

(5)

At this juncture, \( Y_t \) is predictable piece and is equal to stochastic component given on RHS of the equation and \( e' \) stands as the error recognized. An examination of ANNs clarified that the ARX (moving normal autoregressive exogenous) estimating prototypical is utilized by approaches for ancient frameworks idea and rough set (RS) idea to generate an instinctual standard marketplace gauging the portfolio choice game plan and consequences depicted that the combination approach not just gives prevalent forecast exactness than GM procedure, yet in addition harvests predominant pace of profits on specific frameworks (Huang, 2009). Subsequently, we would utilize NARX framework in our exploration to anticipate costs more promptly.

**Proposed Model for Optimizing Portfolios using ANNs**

The returns from ANNs \( \hat{R} \), calculated with ANNs time series forecaster are utilized as anticipated returns in our ANNs version, whilst the risk and return for the portfolio is computed as

\[ ANNs \text{ Portfolio Risk } \sigma^2 = \frac{1}{N} \sum_{t=1}^{N} (R_t - \hat{R})^2 \]  

(6)

\[ ANNs \text{ Portfolio returns } R_p = \sum_{t=1}^{M} X_t \hat{R}_t \]  

(7)

Semi variance = \( \frac{1}{n} \times \sum_{rt<\text{Average}} (\text{Average} - rt)^2 \)  

(8)

Where, \( n \) =The total number of observations below the mean, \( rt \) =The observed value, and \( \text{Average} \) =The mean or target value of the dataset

The extent of interactive risk \( \hat{\gamma}_{ij} \) is expounded as:

\[ \text{Interactive Risk(Covariance)} \hat{\gamma}_{ij} = \frac{1}{N} \sum_{t=1}^{N} (R_{it} - \hat{R}_i)(R_{jt} - \hat{R}_j) \]  

(9)

After updating all the parameters and formulas we are proposing our model as:

\[ \text{Minimize } \hat{V} = \sum_{i=10}^{M} X_i^2 \hat{\upsilon}_i + \sum_{i=10}^{M} \sum_{j=10, i\neq j} X_i X_j \hat{\gamma}_{ij} \]  

(10)

where, \( \sum_{i=10}^{M} X_i \hat{R}_i = R \),

(11)

also \( \sum_{i=10}^{M} X_i = 1 \) and

(12)

and, \( X_{i0} \geq 0, i = 1, ..., M \)

(13)

**Empirical Findings**

Table 1 reports the result of the portfolio risk and portfolio return under equally weighted portfolios i.e. naïve diversification strategy in Pakistan. Furthermore, it reports the portfolio returns and portfolio risk with historical estimation and ANNs-based predictions. Also, we computed the sharp ratio to compare the performance of the portfolio based on historical return estimation with the ANNs predicted returns in Pakistan. Results show that the performance of equally weighted portfolios under historical estimation outperformed the ANNs based on expected return estimations based on a sharp ratio. The sharp ratio under historical estimation results in a value of 0.095 while the sharp ratio under ANNs estimation is 0.0731. Therefore, the study accepts our first hypothesis i.e. the portfolio based on historical return estimation
outperformed the ANNs predicated returns under a naïve diversification strategy.

Table 1. Risk and return for the naïve portfolio

<table>
<thead>
<tr>
<th>Equally-weighted Portfolios</th>
<th>Portfolio Return</th>
<th>Portfolio Risk</th>
<th>Sharp Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hist. Estimation</td>
<td>0.0072</td>
<td>0.0758</td>
<td>0.0950</td>
</tr>
<tr>
<td>ANNs Estimation</td>
<td>0.00624</td>
<td>0.0854</td>
<td>0.0731</td>
</tr>
</tbody>
</table>

Table 2 reports the result of the portfolio risk and portfolio return under mean semi-variance portfolios in Pakistan. Furthermore, it reports the portfolio returns and portfolio risk with historical estimation and ANNs-based predictions. Also, we computed the sharp ratio to compare the performance of the portfolio based on historical return estimation with the ANNs predicated returns in Pakistan. Results show that the performance of mean semi-variance portfolios under ANNs based on expected return estimations outperformed the historical estimation based on a sharp ratio. The sharp ratio under historical estimation results in the value of 0.3464 while the sharp ratio under ANNs estimation is 0.3985. Therefore, the study rejects our second hypothesis i.e. the portfolio based on historical return estimation outperformed the ANNs predicated returns under the mean semi-variance optimization strategy.

Table 2. Risk and return for Mean Semi Variance Portfolios

<table>
<thead>
<tr>
<th>Efficient Portfolios</th>
<th>Portfolio Return</th>
<th>Portfolio Risk</th>
<th>Sharp Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hist. Estimation</td>
<td>0.2914</td>
<td>0.8412</td>
<td>0.3464</td>
</tr>
<tr>
<td>ANNs Estimation</td>
<td>0.3291</td>
<td>0.8259</td>
<td>0.3985</td>
</tr>
</tbody>
</table>

Table 3 reports the result of the portfolio risk and portfolio return under constraint mean semi-variance portfolios in Pakistan. Furthermore, it reports the portfolio returns and portfolio risk with historical estimation and ANNs-based predictions. Also, we computed the sharp ratio to compare the performance of the portfolio based on historical return estimation with the ANNs predicated returns in Pakistan. Results show that the performance of constraint mean semi-variance portfolios under ANNs-based expected return estimations outperformed the historical estimation based on a sharp ratio. The sharp ratio under historical estimation results in the value of 0.3585 while the sharp ratio under ANNs estimation is 0.4385. Therefore, the study rejects our third hypothesis i.e. the portfolio based on historical return estimation outperformed the ANNs predicated returns under constraint mean semi-variance optimization strategy.

Table 3. Risk and return for Constraint Mean Semi Variance Portfolios

<table>
<thead>
<tr>
<th>Efficient Portfolios (Budget Constraint)</th>
<th>Portfolio Return</th>
<th>Portfolio Risk</th>
<th>Sharp Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hist. Estimation</td>
<td>0.2338</td>
<td>0.6521</td>
<td>0.3585</td>
</tr>
<tr>
<td>ANNs Estimation</td>
<td>0.2817</td>
<td>0.6424</td>
<td>0.4385</td>
</tr>
</tbody>
</table>

Table 4 reports the result of the portfolio risk and portfolio return under constraint mean semi-variance portfolios by applying the transaction cost condition in Pakistan. Furthermore, it reports the portfolio returns and portfolio risk with historical estimation and ANNs-based predictions. Also, we computed the sharp ratio to compare the performance of the portfolio based on historical return estimation with the ANNs predicated returns in Pakistan. Results show that the performance of constraint (transaction cost) mean semi-variance portfolios under ANNs based on expected return estimations outperformed the historical estimation based on a sharp ratio. The sharp ratio under historical estimation results in the value of 0.2570 while the sharp ratio under ANNs estimation is 0.2312. Therefore, the study rejects our fourth hypothesis i.e. the portfolio based on historical return estimation outperformed the ANNs predicated returns under constraint
(transaction cost) mean semi-variance optimization strategy.

Table 4. Risk and return for Constraint (transaction cost) Mean Semi Variance Portfolios

<table>
<thead>
<tr>
<th>Efficient Portfolios (Transaction Cost)</th>
<th>Portfolio Return</th>
<th>Portfolio Risk</th>
<th>Sharp Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hist. Estimation</td>
<td>0.1568</td>
<td>0.6782</td>
<td>0.2312</td>
</tr>
<tr>
<td>ANNs Estimation</td>
<td>0.1986</td>
<td>0.7728</td>
<td>0.2570</td>
</tr>
</tbody>
</table>

Conclusion of Study
The challenge of managing a portfolio effectively is allocating capital among numerous stock holdings to achieve maximum profit. The primary purpose of this study is to investigate the part that artificial neural networks (ANNs) play in the process of portfolio optimization of Pakistani companies that are listed on the Pakistan stock exchange (PSX). To pick and optimize a portfolio in the most effective way possible, we made use of the closing stock prices of 50 listed firms that are included in the KSE-100 index. The study time-period starts in January 2017 and ends in December 2021. The data has been collected from the website of the Pakistan stock exchange i.e. the data portal of the Pakistan stock exchange. The objective of this study is to compare the portfolios based on historical return estimation with the ANNs predicated returns under naïve diversification strategy, mean semi-variance optimization strategy, mean semi-variance optimization strategy by setting budget constraints and mean semi-variance optimization strategy by setting transaction cost constraints in Pakistan. We evaluate the portfolios based on their sharp ratio i.e. portfolio return per unit of portfolio risk in Pakistan.

Results show that the performance of equally weighted portfolios under historical estimation outperformed the ANNs-based expected return estimations. Furthermore, we find that the performance of mean semi-variance portfolios, mean semi-variance optimization strategy by setting budget constraints and mean semi-variance optimization strategy by setting transaction cost constraints under ANNs-based expected return estimations outperformed the historical estimation based on the sharp ratio in Pakistan. This is because neural networks suggest explanations to intricate calculations, thereby uncovering patterns from inputs that were previously unexplainable, and serving as a deciding factor within a variety of different situations. ANNs provide an alternative that is more rational to traditional approaches that are traditionally constrained by stringent constraints. Because an ANN is capable of apprehending multiple categories of associations, it enables the manipulator to quickly and relatively easily represent the process, which would otherwise be difficult or impossible to express. The dominant role that ANNs play as the best indicators and the most practical method for portfolio optimization is illustrated by both the Sharpe ratio.

The study suggests that investors, fund managers, and portfolio analysts should focus on the more sophisticated neural network-based choice for the development of portfolios in the equity market of Pakistan. It is possible to conduct comparative research of the various stock exchanges throughout the world to eliminate the possibility of bias and to determine in which countries scenario ANNs have proven to be the most accurate forecasters of stock prices. When it comes to the selection of a portfolio and the optimization of that portfolio, one potential bright spot is the incorporation of additional criteria for analysis, such as dividends and gains (Kolm, 2014).

References


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