Volume and Issues Obtainable at Center for Sustainability Research and Consultancy

Review of Politics and Public Policy in Emerging Economies
ISSN: 2708-3829 & (E): 2708-356X
Volume 2: No. 2, December 2020
Journal homepage: www.publishing.globalcsrc.org/rope

Ramsey Taxation and Public Goods in the Solow-Uzawa Growth Model

***Wei-Bin Zhang,** Ritsumeikan Asia Pacific University, Japan

*Corresponding author's email: wbz1@apu.ac.jp

ARTICLE DETAILS

History
Revised format: Nov 2020
Available Online: Dec 2020

Keywords
Uzawa's model, Ramsey taxation, optimal tax rates, Solow model, public good

JEL Classification
H2, H71

ABSTRACT

Purpose: This paper examines issues related to optimal taxation similar for those addressed by Ramsey in his celebrated 1927 paper. This model determines optimal taxation to maximize utility with revenue as endogenous variable. Optimal taxation is re-examined within theoretical neoclassical growth framework. We re-construct the Solow-Uzawa two-sector model with optimal taxation. The economy is built with an economic structure of the public, capital goods and consumer goods sectors. Public goods enter the utility function. The government receives tax revenue from consumption of consumer goods and capital goods. The government revenue is epnt on the public sector.
Methodology: Comparative analysis is conducted to analyze how the tax rates and other economic variables are endogenously changed owing to exogenous changes in some parameters.
Findings: We find out the optimal taxation rule in the national economy. The model describes nonlinear dynamic interactions between economic structural change, macroeconomic growth, capital accumulation, and optimal tax rates with perfect competition in input and private goods markets.
Implications: The topic is also practically important as governments in many market economies have made extensive public inventions after WWII. This new trend has caused economists to theoretically and empirically analyze taxation in various economic systems.

Recommended citation: Zhang, W. B. (2020). Ramsey Taxation and Public Goods in the Solow-Uzawa Growth Model. *Review of Politics and Public Policy in Emerging Economies, 2* (2) 97-110.

Introduction

In his paper on evaluating Ramsey's contribution to economics, Stiglitz (2015: 235) makes the following judgement: "Frank Ramsey's brilliant 1927 paper, modestly entitled, 'A contribution to the theory of taxation', is a landmark in the economics of public finance. Nearly a half century later, through the work of Diamond and Mirrlees (1971) and Mirrless (1971), his paper can be thought of as launching the field of optimal taxation and revolutionizing public finance." Ramsey (1927:47) states what he is concerned about in his seminal paper: "a given revenue is to be raised by proportionate taxes on some or all uses of income, the taxes on different uses being possible at different rates; how should these rates be adjusted in order that the decrement of utility may be minimum?" Like Ramsey, this paper is also concerned with optimal taxation in an economy with a homogeneous population and a source of the government's taxes solely on commodities. But we deviate from Ramsey's approach in multiple ways. In our approach optimal taxation is determined not with fixed revenue, but to maximize the representative household's utility. The tax income or revenue is an endogenous variable in our model. Different from the Ramsey model, our model is developed within neoclassical framework. It is especially framed with the Solow-Uzawa growth model of wealth accumulation.

Since Ramsey's 1927 pioneering work on optimal taxation, the topic has caused attention of many researchers in different fields of economics. There appear many formal analytical works on optimal taxation under different institutions. The topic is also practically important as governments in many market economies have made extensive public inventions after WWII. This new trend has caused economists to theoretically and empirically analyze taxation in various economic systems. In the literature of optimal tax theory, one finds various feasible taxes and their different implications for different households. The government has various objectives when taxation is made (e.g., Diamond and Mirrlees, 1971, 1971a; Wildasin, 1988; Slemrod, 1990; and Wilson and Gordon, 2003). Tax configurations have close relations to their operational efficiencies and welfare implications (e.g., Auerbach, 1985; Zodrow and Mieszkowski, 1986; Stiglitz, 1987; and Lai, 2019). Researchers have made great efforts in analyzing optimal taxation. Nevertheless, there is a main limitation in most of these studies. Analytical works are mostly conducted within static equilibrium frameworks. It is apparently imperative to determine optimal taxation within a dynamic theory. The study is carried out on the basis of the well-known Solow-Uzawa two-sector neoclassical growth model by applying Zhang's alternative approach to behavior of household (Zhang, 2005, 2008, 2020).

Rather than determining taxes on commodities to minimize the decrement of utility with fixed revenue as in the Ramsey approach, we determine optimal taxation to maximize utility with revenue as endogenous variable. Optimal taxation is determined for an economy described by neoclassical growth theory. The main economic structure is framed by the Solow-Uzawa two-sector model. A public sector is integrated into the model. The economy consists of the capital goods, public, and consumer goods sectors. The utility function is directly affected by public goods. The government's revenue comes from taxing consumption of consumer goods and capital goods. The revenue is used up for supporting the public sector. We find the optimal taxation rule for the national economy. The model describes nonlinear dynamic interactions between macroeconomic growth, accumulation of wealth/capital, economic structural change, and optimal tax rates. The rest paper is constructed with 4 sections. Section 2 develops the dynamic framework for analyzing endogenous public goods with optimal taxes on capital goods and consumer goods. Section 3 finds a computational procedure to simulate the dynamics and identifies the economic equilibrium point. Section 4 studies comparative analysis with regards to a few of the coefficients. Section 5 concludes the study. The Appendix establishes the main results of Section 3.

The Solow-Uzawa Growth Model with Taxes on Consumption of two Goods

The model is framed on the basic structure of the Solow-Uzawa two-sector neoclassical model (Solow, 1956; and Uzawa, 1961) and the Ramsey model with taxation (Ramsey, 1927). It also integrates an alternative approach to the public sector and the public goods (Zhang, 2016). This paper generalizes Zhang's approach in that tax rates are considered endogenous variables like in the Ramsey model. We follow Ramsey (1927:47): "I propose to neglect altogether questions of distribution and considerations arising from differences in the marginal utility of money to different people; and I shall deal only with a purely competitive system with no foreign trade." The economy thus has three sectors. Two private sectors are perfectly competitive and are framed as in the traditional two - consumer goods and capital goods - sectors growth model. The other sector is the public sector. The government's tax revenue is spent on the public sector. Capital and labor are mobile between the sectors. Assets are owned by the households. There is constant and homogeneous population. Let subscript index, p, s, and i stand for the public goods sector, the consumer goods sector, and the capital goods sector. We use $K_j(t)$ and $N_j(t)$ to represent the inputs of capital stocks and labor force by sector j. Let $F_j(t)$ mean the output level of sector j.

The Capital Goods Sector
The sector's production function is:

$$F_i(t) = A_i\, K_i^{\alpha_i}(t)\, N_i^{\beta_i}(t),\ \ \alpha_i, \beta_i\ >\ 0,\ \ \alpha_i\ +\ \beta_i\ =\ 1,\ \ (1)$$

where A_i, α_i, and β_i are parameters. Variables $w(t)$ and $r(t)$ are respectively the wage rate and interest rate. We use δ_k to denote depreciation rate of capital goods. The profit is:

$$\pi_i(t)\ =\ F_i(t) - (r(t) + \delta_k)r(t) - w(t)N_i(t).$$

We maximize the profit and get the marginal conditions:

$$r_\delta(t)\ =\ \frac{\alpha_i\, F_i(t)}{K_i(t)},\ \ w(t)\ =\ \frac{\beta_i\, F_i(t)}{N_i(t)},\ \ (2)$$

where $r_\delta(t)\ \equiv\ r(t)\ +\ \delta_k$.

The consumer goods sector
The consumer goods sector's production function is:

$$F_s(t)\ =\ A_s\, K_s^{\alpha_s}(t)\, N_s^{\beta_s}(t),\ \ \alpha_s,\ \beta_s\ >\ 0,\ \ \alpha_s\ +\ \beta_s\ =\ 1,\ \ (3)$$

where α_s and β_s are parameters. The marginal conditions are:

$$r_\delta(t)\ =\ \frac{\alpha_s\, p(t)\, F_s(t)}{K_s(t)},\ \ w(t)\ =\ \frac{\beta_s\, p(t)\, F_s(t)}{N_s(t)}.\ \ (4)$$

The Public Sector
We follow Ramsey's assumption that the government raises revenue through taxing only on commodities. This strict assumption is obviously limited as contemporary governments in capitalist economies try to tax any taxable sources which they can find and justify. We construct the public sector which is financially supported by the government's tax income. The public sector pays the capital stocks and workers the equal rates to those in the capital and labor markets. The government utilizes the two input factors in efficiency. The sector uses the government's support to maximize public services. The public sector supplies services by using capital $K_p(t)$ and labor force $N_p(t)$ with the following supply function:

$$F_p(t)\ =\ A_p\, K_p^{\alpha_{0p}}(t)\, N_p^{\beta_{0p}}(t),\ \ \alpha_{0p}, \beta_{0p}, A_p\ >\ 0.$$

The budget constraint of the public sector is:

$$w(t)\, N_p(t)\ +\ r_\delta(t)\, K_p(t)\ =\ Y_p(t). \qquad (5)$$

Maximizing $F_p(t)$ under (5) yields:

$$r_\delta(t)\,K_p(t) \;=\; \alpha_p\,Y_p(t),\;\; w(t)\,N_p(t) \;=\; \beta_p\,Y_p(t),\;\; (6)$$

in which

$$\alpha_p \;\equiv\; \frac{\alpha_{0p}}{\alpha_{0p} \;+\; \beta_{0p}},\;\; \beta_p \;\equiv\; \frac{\beta_{0p}}{\alpha_{0p} \;+\; \beta_{0p}}.$$

Behavior of the Household

This study applies Zhang's utility approach for modeling consumers' behavior. The approach is discussed at length by Zhang (2005, 2008, 2020). Let $\bar{k}(t)$ the representative household's value of wealth. Let h stand for human capital. The current income is the wage and interest incomes:

$$y(t) \;=\; r(t)\,\bar{k}(t) \;+\; h\,w(t).\;\; (7)$$

The sum of the value of the household's wealth and the current income is called disposable income:

$$\hat{y}(t) \;=\; y(t) \;+\; \bar{k}(t).\;\; (8)$$

The disposable income is divided between consumption and saving. Substituting (8) into (9) yields:

$$\hat{y}(t) \;\equiv\; R(t)\bar{k}(t) + hw(t),\;\; (9)$$

where $R(t) \;\equiv\; 1 \;+\; r(t)$. The household distributes the disposable income between consumer goods $c_s(t)$, capital goods $c_i(t)$, and saving $s(t)$. We use $\tau_i(t)$ and $\tau_s(t)$ to stand for tax rates on consumption of capital goods and consumer goods and introduce $\bar{\tau}_x(t) \;\equiv\; 1 \;+\; \tau_x(t)$. The budget constraint implies:

$$\bar{\tau}_s(t)\,p(t)\,c_s(t) \;+\; \bar{\tau}_i(t)\,c_i(t) \;+\; s(t) \;=\; \hat{y}(t).\;\; (10)$$

Utility level $U(t)$ of the representative household is related to the public goods $F_p(t)$, $c_s(t)$, $c_i(t)$ and $s(t)$ as follows:

$$U(t) \;=\; \theta\,F_p^{d_0}(t)\,s^{\lambda_0}(t)\,\left(c_s^{\xi_0}(t) \;+\; \xi\,c_i^{\xi_0}(t)\right)^{1/\xi_0},\;\; d_0,\,\gamma_0,\xi,\lambda_0 > \;0,\;\; (11)$$

where the power coefficients are the utility elasticities of the variables of the representative household. The household makes the decision, taking the tax rates as given. As shown in Appendix A.1, maximizing $U(t)$ subject to (11) yields:

$$s(t) \;=\; \frac{\lambda_0\,\hat{y}(t)}{1+\lambda_0},\;\; c_i(t) = \frac{\xi\,\hat{y}(t)}{(1+\lambda_0)\,(P^{\xi_0}(t) \;+\; \xi)\,\bar{\tau}_i(t)},$$

$$c_s(t) = \frac{\xi\,P(t)\,\hat{y}(t)}{(1+\lambda_0)\,(P^{\xi_0}(t) \;+\; \xi)\,\bar{\tau}_i(t)},\;\; (12)$$

where

$$P(t) \equiv \left(\frac{\xi \, \bar{\tau}_s(t) \, p(t)}{\bar{\tau}_i(t)}\right)^{\xi_4}, \quad \xi_4 \equiv \frac{1}{\xi_0 - 1}.$$

The change of wealth minus the saving is the dissaving The definition of $s(t)$ yields the following wealth accumulation equation for the representative household:

$$\dot{\bar{k}}(t) = s(t) - \bar{k}(t). \quad (13)$$

The Government's Tax Income and Planned Expenditure

The government's tax income is from taxing consumption of two goods. We have the following government revenue:

$$T_p(t) = \tau_i(t) \, c_i(t) \, \bar{N} + \tau_s(t) \, p(t) \, c_s(t) \, \bar{N}. \quad (14)$$

Endogenous Tax Rates for Maximizing Welfare

It is assumed that the government optimizes the tax rates to maximize the utility function. By (11) and (12):

$$U(t) = \frac{\xi \, \theta}{1 + \lambda_0} \left(\frac{\lambda_0}{1 + \lambda_0}\right)^{\lambda_0} \frac{F_p^{d_0}(t) \, \hat{y}^{1+\lambda_0}(t)}{\bar{\tau}_i(t)} \left(P^{\xi_0}(t) + \xi\right)^{\xi_1}, \quad (15)$$

where $\xi_1 \equiv (1 - \xi_0)/\xi_0$. From (9) and (7), we have:

$$F_p(t) = A_p \left(\frac{\alpha_p}{r_\delta(t)}\right)^{\alpha_{0p}} \left(\frac{\beta_p}{w(t)}\right)^{\beta_{0p}} Y_p^{\alpha_{0p}+\beta_{0p}}(t). \quad (16)$$

Insert (17) in (18):

$$U(t) = \frac{u(t) \left(P^{\xi_0}(t) + \xi\right)^{\xi_1} Y_p^d(t)}{\bar{\tau}_i(t)}, \quad (17)$$

where $d \equiv \left(\alpha_{0p} + \beta_{0p}\right) d_0$ and

$$u(t) \equiv A_p^{d_0} \left(\frac{\alpha_p}{r_\delta(t)}\right)^{d_0 \, \alpha_{0p}} \left(\frac{\beta_p}{w(t)}\right)^{d_0 \, \beta_{0p}} \frac{\xi \, \theta}{1 + \lambda_0} \left(\frac{\lambda_0}{1 + \lambda_0}\right)^{\lambda_0} \hat{y}^{1+\lambda_0}(t).$$

As the government's tax income is equal to the costs of the public sector, we have:

$$Y_p(t) = T_p(t) = \tau_i(t) \, c_i(t) \, \bar{N} + \tau_s(t) \, p(t) \, c_s(t) \, \bar{N}. \quad (18)$$

By (10) and (19), $U(t)$ is represented as a function of $\tau_s(t)$ and $\tau_i(t)$:

$$U(t) = \frac{u(t) \left(P^{\xi_0}(t) + \xi\right)^{(1-\xi_0)/\xi_0} \bar{N}^d \left(\tau_i(t) \, c_i(t) + \tau_s(t) \, p(t) \, c_s(t)\right)^d}{\bar{\tau}_i(t)}. \quad (19)$$

The government makes the decision, taking the levels of consumption as given. Maximizing $U(t)$ with $\tau_i(t)$ and $\tau_s(t)$ as variables. The first-order conditions imply:

$$\frac{dc_i(t)}{\tau_i(t)c_i(t) + \tau_s(t)p(t)c_s(t)} - \frac{1}{\bar{\tau}_i(t)} + \frac{P^{\xi_0}(t)}{(P^{\xi_0}(t)+\xi)\bar{\tau}_i(t)} = 0,$$
$$\frac{d\ p(t)\, c_s(t)}{\tau_i(t)c_i(t) + \tau_s(t)p(t)c_s(t)} - \frac{P^{\xi_0}(t)}{(P^{\xi_0}(t) + \xi)\bar{\tau}_s(t)} = 0. \ (20)$$

From (20), we have:

$$\frac{c_s(t)}{c_i(t)} = P(t).$$

We solve the above equation

$$\frac{\bar{\tau}_s(t)}{\bar{\tau}_i(t)} = f\big(c_i(t), c_s(t), p(t)\big) \equiv \left(\frac{c_s(t)}{c_i(t)}\right)^{\xi_0-1} \frac{1}{\xi\, p(t)}. \ (21)$$

It should be noted that as shown by Ramsey (1927; see also Mas-Colell, *et al*. 1995) the optimal taxes lead to every good to get the same proportional reduction in compensated demand. This is what the so-called Ramsey rule implies. From (20) and (22) we solve:

$$\tau_s(t) = \left(\left(\frac{1}{f(t)} - 1\right)P^{\xi_0-1}(t) - \left(P^{\xi_0}(t) + \xi\right) d\, p(t)\right)\bar{\mathrm{f}}(t),$$
$$\tau_i(t) = \frac{1 + \tau_s(t)}{f(t)} - 1. \ (22)$$

where we also use

$$\bar{\mathrm{f}}\Big(\big(c_i(t), c_s(t), p(t)\big)\Big) \equiv \left(\left((d-1)\, P^{\xi_0}(t) + d\, \xi\right) p(t) - \frac{P^{\xi_0-1}(t)}{f(t)}\right)^{-1}.$$

Equilibrium of the two Goods

Consumer goods market in equilibrium satisfies:

$$c_s(t)\bar{N} = F_s(t). \ (23)$$

The equilibrium condition in capital goods market implies:

$$c_i(t)\bar{N} + s(t)\bar{N} + \delta_k K(t) = F_i(t) + K(t). \ (24)$$

We mean that in equilibrium the capital stock change and capital goods consumed is equal to capital goods produced minus the capital depreciated.

Capital and Labor being Fully Employed

The national capital stock $K(t)$ is fully employed by the three sectors. Labor and capital are fully employed:

$$K_i(t) + K_s(t) + K_p(t) = K(t), \ (25)$$

$$N_i(t) + N_s(t) + N_p(t) = h\,\bar{N}. \;(26)$$

The wealth is held by the households. We thus have:

$$K(t) = \bar{k}(t)\,\bar{N}. \;(27)$$

The construction of the dynamic growth model is completed.

The Economic Dynamics

The Appendix gives a computational procedure about how to follow the motion of the economic system starting from a single differential equation. The variable $z(t)$ of the equation is $z(t) \equiv w(t)/(r(t) + \delta_k)$. The equation and procedure are summarized in the following lemma.

Lemma

The motion of $z(t)$ follows the following differential equation:

$$\dot{z}(t) = H\big(z(t)\big), \;(28)$$

in which $H(t)$ is given in the Appendix. Al the other variables are given as functions of $z(t)$ by the following procedure: $\bar{k}(t)$ by (A2.14) $\rightarrow$ $r(t)$ and $w(t)$ by (A2.2) $\rightarrow r_\delta(t) = r(t) - \delta_k \rightarrow$ $p(t)$ by (A2.3) $\rightarrow c_s(t)$ and $c_i(t)$ by (A2.13) $\rightarrow \tau_s(t)$ by (22) $\rightarrow \tau_i(t)$ by (22) $\rightarrow \hat{y}(t)$ by (10) $\rightarrow K_p(t)$ by (A2.10) $\rightarrow K_s(t)$ by (A2.7) $\rightarrow K(t) = \bar{k}(t)N \rightarrow K_i(t)$ by (A2.6) $\rightarrow N_j(t)$, $j = i, s, p \rightarrow F_j(t)$ by the defined functions $\rightarrow U(t)$ by (19) $\rightarrow Y_p(t)$ by (5) $\rightarrow T_p(t) = Y_p(t)$.

The expressions are very tedious. The rest of the study is limited to the equilibrium economic structure. To simulate the economic equilibrium structure, we choose the parameter values:

$$\bar{N} = 50,\; A_i = 1,\; A_s = 2.5,\; A_p = 0.9,\; \alpha_i = 0.32,\; \alpha_s = 0.34,\; \alpha_{0p} = 0.2,\; \beta_{0p} = 0.5,\; \lambda_0 = 4,\; \xi_0 = 0.55,\; \xi = 2,\; \theta = 1,\; \omega = 1,\; h = 3,\; d_0 = 0.2,\; \delta_k = 0.04. \;(29)$$

The population is 50. The propensity to save λ_0 is 4. The propensity to consume the aggregated goods ξ_0 is 0.1. The share of capital goods is 2, while the share of consumer goods is 1. The depreciation rate is chosen at 4 percent. The choice of the parameter values is not based on a special economy. This will not affect the basic purpose to give some insights into how different parameter values affect the tax rates and economic equilibrium.

Following the lemma, we straightforwardly have the equilibrium values of the variables:

$$\tau_i = 0.097,\; \tau_s = 0.302,\; Y = 268.6,\; Y_p = 32.4,\; K = 926.2,$$
$$F_i = 172.6,\; F_s = 165,\; F_p = 9.86,\; N_i = 96.5,\; N_s = 34.5,\; N_p = 19,$$
$$K_i = 594,\; K_s = 232.6,\; K_p = 99.6,\; r = 0.053,\; p = 1.22,\; w = 0.39,$$

$\hat{y} = 23.15, \ \bar{k} = 18.5, \ c_i = 2.71, \ c_s = 3.3, \ U = 3978180.$ (30)

The tax rate on consumer goods is higher than the tax rate on capital goods. Most of the labor force and capital stock are employed by the capital goods sector. In (30), the variable $Y(t)$ measures the national income:

$$Y(t) = (r(t) + \delta_k)\, K(t) + w(t)\, h\, \bar{N}.$$

We now analyze effects of exogenous changes on the system.

The Impact of Exogenous Changes in Some Parameters

The previous section identified an equilibrium point. This implies that we can easily analyze how exogenous changes in parameters have effects on the equilibrium tax rates and equilibrium economic structure. We use $\bar{\Delta}x$ express the variable x's change rate the in percentage caused by due to some exogenous changes in parameter. For convenience of expression, we call consumer goods as good 1, capital goods as good 2, and public goods as good 3.

A Change in Good 3's Utility Elasticity

Consider a rise of the good 3's utility elasticity: $d_0 = 0.2$ to 0.205. The simulation result is listed in (31). As the utility elasticity for good 3 is increased, the tax rates on good 1 and good 2 are enhanced. The public sector is expanded. More public services are provided. The public sector's uses of capital and labor force are increased. The consumer consumes less good 1 and good 2. The interest rate and wage rate are slightly reduced. The national income and wealth are almost not changed. The two private sectors are shrunken. They produce less and use less inputs. The price of good 1 is slightly increased. The utility level is enhanced. We see that if the society "appreciates" more good 3 and public sector is effectively operated, the government will make more intervention, enhancing the people's welfare, even though the GDP is slightly affected.

$$\bar{\Delta}\tau_i = 4.61, \ \bar{\Delta}\tau_s = 1.76, \ \bar{\Delta}Y \approx 0, \ \bar{\Delta}Y_p = 2.5, \ \bar{\Delta}K \approx 0, \ \bar{\Delta}F_i = -0.32,$$
$$\bar{\Delta}F_s = -0.41, \ \bar{\Delta}F_p = 1.74, \ \bar{\Delta}N_i = -0.34, \ \bar{\Delta}N_s = -0.43, \ \bar{\Delta}N_p = 2.48,$$
$$\bar{\Delta}K_i = -0.28, \ \bar{\Delta}K_s = -0.37, \ \bar{\Delta}K_p = 2.54, \ \bar{\Delta}r = -0.07, \ \bar{\Delta}p = 0.02,$$
$$\bar{\Delta}w = -0.001, \ \bar{\Delta}\hat{y} \approx 0, \ \bar{\Delta}\bar{k} \approx 0, \ \bar{\Delta}c_i = -0.41, \ \bar{\Delta}c_s = -0.41, \ \bar{\Delta}U = 1.1. \ (31)$$

The Output Elasticity of Labor Force in Good 3 Supply being Increased

We now examine how the system is affected if $\alpha_{0p} = 0.2$ to 0.205. We get the simulation result in (32). The tax rates on good 1 and good 2 are enhanced. The public sector is expanded. The public sector's inputs are enhanced. The consumer consumes less good 1 and good 2. The interest rate and wage rate are slightly reduced. The national income and wealth are almost not changed. The two private sectors are shrunken. They produce less and use less inputs. The price of good 1 is slightly increased. The utility level is enhanced. We see that a more effective use of physical capital has directionally a similar impact on the economic system when the utility elasticity for good 3 is increased as discussed above.

$$\bar{\Delta}\tau_i = 1.33, \ \bar{\Delta}\tau_s = 0.49, \ \bar{\Delta}Y = 0.003, \ \bar{\Delta}Y_p = 0.71, \ \bar{\Delta}K = 0.0003,$$
$$\bar{\Delta}F_i = -0.09, \ \bar{\Delta}F_s = -0.12, \ \bar{\Delta}F_p = 2.9, \ \bar{\Delta}N_i = -0.3, \ \bar{\Delta}N_s = -0.3, \ \bar{\Delta}N_p = 2.3,$$
$$\bar{\Delta}K_i = -0.01, \ \bar{\Delta}K_s = -0.03, \ \bar{\Delta}K_p = 0.09, \ \bar{\Delta}r = 0.32, \ \bar{\Delta}p = -0.09, \ \bar{\Delta}w = 0.005,$$
$$\bar{\Delta}\hat{y} = \bar{\Delta}\bar{k} = 0.0003, \ \bar{\Delta}c_i = -0.12, \ \bar{\Delta}c_s = -0.12, \ \bar{\Delta}U = 0.45. \ (32)$$

The Saving Propensity is Increased

Let us study the impact of the following change: $\lambda_0 = 4$ to 4.1. We get the simulation result in (33). The tax rate on good 2 is reduced in association in the rise in the saving propensity. The tax rate on good 1 is augmented. The public sector gets more income. More good 3 is provided. The public sector employs more capital but less labor force. Consumption levels of good 1 and consumer good 2 are enhanced. The interest rate and wage rate fall. The price of good 1 rises. The national income and wealth are increased. The two private sectors produce more. The consumer goods sector employs less labor force but more capital. The household has more disposable income and wealth. The utility level is enhanced.

$$\bar{\Delta}\tau_i = -0.23,\ \bar{\Delta}\tau_s = 0.18,\ \bar{\Delta}Y = 1,\ \bar{\Delta}Y_p = 0.65,\ \bar{\Delta}K = 3.2,$$
$$\bar{\Delta}F_i = 1.21,\ \bar{\Delta}F_s = 0.68,\ \bar{\Delta}F_p = 0.39,\ \bar{\Delta}N_i = 0.21,\ \bar{\Delta}N_s = -0.39,$$
$$\bar{\Delta}N_p = -0.34,\ \bar{\Delta}K_i = 3.4,\ \bar{\Delta}K_s = 2.8,\ \bar{\Delta}K_p = 2.8,\ \bar{\Delta}r = -3.7,$$
$$\bar{\Delta}p = 1,\ \bar{\Delta}w = -0.06,\ \bar{\Delta}\hat{y} = 2.7,\ \bar{\Delta}\bar{k} = 3.2,\ \bar{\Delta}c_i = \bar{\Delta}c_s = 0.68,\ \bar{\Delta}U = 53.4. \quad (33)$$

The Productivity of Good 2 Sector being Increased

The total factor productivity of the capital goods sector is enhanced: $A_i = 1$ to 1.05. We get the simulation result in (34). The tax rate on good 2 becomes higher but the tax rate on good 1 becomes lower. The public sector gets more tax income. The national income and national wealth are enhanced. More public services are provided. The public sector increases labor force and capital. The consumer consumes more good 1 and good 2. The household has more disposable income and wealth. The wage rate and interest rate are enhanced. The two private sectors are expanded. The price of good 1 is increased. The utility level is enhanced.

$$\bar{\Delta}\tau_i = 17.5,\ \bar{\Delta}\tau_s = -13.6,\ \bar{\Delta}Y = \bar{\Delta}Y_p = \bar{\Delta}K = 7.44,$$
$$\bar{\Delta}F_i = 6.2,\ \bar{\Delta}F_s = 5.8,\ \bar{\Delta}F_p = 1.45,\ \bar{\Delta}N_i = -1.18,\ \bar{\Delta}N_s = 3.3,\ \bar{\Delta}N_p = 0.02,$$
$$\bar{\Delta}K_i = 6.1,\ \bar{\Delta}K_s = 10.9,\ \bar{\Delta}K_p = 7.4,\ \bar{\Delta}r = 0.08,\ \bar{\Delta}p = 7.4,\ \bar{\Delta}w = 4.85,$$
$$\bar{\Delta}\hat{y} = \bar{\Delta}\bar{k} = 7.44,\ \bar{\Delta}c_i = \bar{\Delta}c_s = 5.8,\ \bar{\Delta}U = 41.4. \quad (34)$$

The Consumer Goods Sector Increases its Productivity

Let us study a rise in the total factor productivity of the consumer goods sector: $A_s = 2.5$ to 2.6. We get the simulation result in (35). The tax rate on good 2 is reduced but the tax rate on good 1 is enhanced. There is no impact on the public sector's tax income. The national income and wealth are almost not affected. Good 3 is slightly reduced. The consumer consumes more consumer goods and capital goods. The household has almost the same disposable income and wealth. The wage rate and interest rate are reduced. The two private sectors are expanded. The capital sector employs more labor force and capital. The consumer goods sector employs less labor force and capital. The price of good 1 is increased. The utility level is enhanced.

$$\bar{\Delta}\tau_i = -13.88,\ \bar{\Delta}\tau_s = 11.7,\ \bar{\Delta}Y = \bar{\Delta}Y_p = \bar{\Delta}K \approx 0,\ \bar{\Delta}F_i = 0.98,\ \bar{\Delta}F_s = 1.24,$$
$$\bar{\Delta}F_p = -0.001,\ \bar{\Delta}N_i = 0.96,\ \bar{\Delta}N_s = -2.7,\ \bar{\Delta}N_p = -0.02,\ \bar{\Delta}K_i = 1.01,$$
$$\bar{\Delta}K_s = -2.6,\ \bar{\Delta}K_p = 0.04,\ \bar{\Delta}r = -0.07,\ \bar{\Delta}p = 0.02,\ \bar{\Delta}w = -3.85,\ \bar{\Delta}\hat{y} = \bar{\Delta}\bar{k} \approx 0,$$
$$\bar{\Delta}c_i = \bar{\Delta}c_s = 1.24,\ \bar{\Delta}U = 1.24. \quad (35)$$

We also analyzed the impact of exogenous changes in other parameters. We made the following change: $\xi_0 = 0.55$ to 0.56. The utility level is reduced. The rest variables are not affected. We also made the following change: $\xi = 2$ to 2.1. The tax rate on capital goods is enhanced. The tax rate on the consumer goods is enhanced. The utility level is increased. The rest variables are not affected. Make a rise in A_p. The good 3 supply and utility level are enhanced. The rest variables are not changed. We changed the population. The macroeconomic real variables are proportionally increased, while the rest variables are not affected. We increased the population. The macroeconomic and microeconomic real variables and utility level are increased. The tax rates and the prices are not affected.

Conclusions

This paper examined some issues related to optimal taxation similar to those addressed by Ramsey in his celebrated 1927 paper. Rather than determining taxes on commodities to minimize the decrement of utility with given revenue in the Ramsey approach, we determine optimal taxation to maximize utility with revenue as endogenous variable. We analyzed optimal taxation in an alternative framework. It integrated a few well-developed approaches in economic growth theory. Zhang's public sector was introduced to the Solow-Uzawa model. The economy consists of the public sector, consumer goods sector, and consumer goods sector. Public goods directly enter the utility function. The government gets revenue from taxing consumption of capital goods and consumer goods. The government supports the public sector with the revenue. We got the optimal taxation rule for the national economy. We carried out comparative statics analysis with regards to exogenous changes in a few coefficients. It should be remarked that there is a large amount of the literature on optimal taxation. We also have many studies on spatial tax competition (e.g., Wilson, 1986; Baldwin and Krugman, 2004; and Ihori and Yang, 2009). We can further pursue the research on basis of the literature on neoclassical growth theory and interdependence between fiscal policy (Barro, 1990; Turnovsky, 2004; Gómez, 2008).

Appendix

A1: Solving the Consumer Problem

We now optimize utility (12) subject to (11). We form the Lagrangian function as follows:

$$L = \theta F_p^{d_0} s^{\lambda_0} \left(c_s^{\xi_0} + \xi c_i^{\xi_0}\right)^{1/\xi_0} + b\,(\hat{y} - \bar{\tau}_s p\, c_s - \bar{\tau}_i c_i - s). \quad (A1.1)$$

Maximizing L, we get

$$\frac{\partial L}{\partial c_s} = \frac{c_s^{\xi_0 - 1}}{c_s^{\xi_0} + \xi c_i^{\xi_0}} - \frac{\bar{\tau}_s p\, b}{U} = 0, \;\; j = 1, 2, \quad (A1.2)$$

$$\frac{\partial L}{\partial c_i} = \frac{\xi c_i^{\xi_0 - 1}}{c_s^{\xi_0} + \xi c_i^{\xi_0}} - \frac{\bar{\tau}_i b}{U} = 0, \quad (A1.3)$$

$$\frac{\partial L}{\partial s} = \frac{\lambda_0}{s} - \frac{b}{U} = 0, \quad (A1.4)$$

$$\frac{\partial L}{\partial b} = \hat{y} - \bar{\tau}_s p\, c_s - \bar{\tau}_i c_i - s = 0. \quad (A1.5)$$

From (A1.2) and (A1.3) we have

$$\frac{c_s}{c_i} = P \equiv \left(\frac{\xi \bar{\tau}_s p}{\bar{\tau}_i}\right)^{\xi_4}, \quad (A1.6)$$

where $\xi_4 \equiv 1/(\xi_0 - 1)$.

Insert (A1.6) in (A1.3)

$$\frac{U}{b} = \frac{c_i \bar{\tau}_i \left(P^{\xi_0} + \xi\right)}{\xi}. \quad (A1.7)$$

From (A1.4)-(A1.7) we solve:

$$\frac{U}{b} = \frac{\hat{y}}{1+\lambda_0}. \quad (A1.8)$$

From (A1.8) and (A1.4), we have:

$$s = \frac{\lambda_0\,\hat{y}}{1+\lambda_0}, \quad (A1.9)$$

$$c_i = \frac{\xi\,\hat{y}}{(1+\lambda_0)\,(P^{\xi_0} + \xi)\,\bar{\tau}_i}, \quad (A1.10)$$

$$c_s = \frac{\xi\,P\,\hat{y}}{(1+\lambda_0)\,(P^{\xi_0} + \xi)\,\bar{\tau}_i}. \quad (A1.11)$$

Appendix A2: Verifying the Lemma

From (3) and (17) we get

$$z \equiv \frac{r+\delta_k}{w} = \frac{\bar{\beta}_x\,N_x}{K_x}, \quad x = i, s, p, \quad (A2.1)$$

where $\bar{\beta}_x \equiv \alpha_x/\beta_x$. By (2) we have:

$$r(z) = \alpha_i\,A_i\left(\frac{z}{\bar{\beta}_i}\right)^{\beta_i} - \delta_k. \quad (A2.2)$$

From (A2.1), we have:

$$w = \frac{r+\delta_k}{z}. \quad (A2.3)$$

With (A1) and (4), we get:

$$p = \frac{w}{\beta_s\,A_s}\left(\frac{z}{\bar{\beta}_s}\right)^{\alpha_s}. \quad (A2.4)$$

Substitute (A2.1) into (26):

$$K_i + \frac{\bar{\beta}_i\,K_s}{\bar{\beta}_s} + \frac{\bar{\beta}_i\,K_p}{\bar{\beta}_p} = \frac{\bar{\beta}_i\,h\,\bar{N}}{z}. \quad (A2.5)$$

From (A2.5) and $K_i = K - K_s - K_p$, we have:

$$K_p = \frac{\alpha_x\,\bar{\beta}_i\,h\,\bar{N}}{z} - \alpha_x\,K - \alpha_{x1}\,K_s\,,$$
$$K_i = K - K_s - K_p, \quad (A2.6)$$

where

$$\alpha_x \equiv \left(\frac{\bar{\beta}_i}{\bar{\beta}_p} - 1\right)^{-1}, \quad \alpha_{x1} \equiv \left(\frac{\bar{\beta}_i}{\bar{\beta}_s} - 1\right)\alpha_x.$$

From (12) and (23), we have:

$$K_s = \tilde{f}_0\,\hat{y}, \quad (A2.7)$$

where

$$\tilde{f}_0(z, \bar{\tau}_s, \bar{\tau}_i) \equiv \frac{\xi\, P\, \bar{N}}{(1+\lambda_0)\,(P^{\xi_0} + \xi)\, A_s\, \bar{\tau}_i}\left(\frac{\bar{\beta}_s}{z}\right)^{\beta_s}.$$

By (6) and (19), we have:

$$\left(\frac{w\, z}{\bar{\beta}_p} + r_\delta\right)\frac{K_p}{\bar{N}} = \tau_i\, c_i + \tau_s\, p\, c_s, \quad (A2.8)$$

where we use (A2.1). By (10) and (12), we have

$$\tau_s\, p\, c_s + \tau_i\, c_i = \left(1 - \frac{\xi(1 + p\, P)}{(P^{\xi_0} + \xi)\, \bar{\tau}_i}\right)\frac{\hat{y}}{1+\lambda_0}. \quad (A2.9)$$

Insert (A2.9) in (A2.8)

$$K_p = \tilde{f}\, \hat{y}, \quad (A2.10)$$

where

$$\tilde{f}(z, c_s, c_i) \equiv \left(1 - \frac{\xi(1 + p\, P)}{(P^{\xi_0} + \xi)\, \bar{\tau}_i}\right)\left(\frac{w\, z}{\bar{\beta}_p} + r_\delta\right)^{-1}\frac{\bar{N}}{1+\lambda_0}.$$

Insert (A2.10) and (A2.7) in (A2.6)

$$\left(\tilde{f} + \alpha_{x1}\, \tilde{f}_0\right)\hat{y} = \frac{\alpha_x\, \bar{\beta}_i\, h\, \bar{N}}{z} - \alpha_x\, K. \quad (A2.11)$$

Insert (27) and (9) in (A11)

$$\bar{k}(z, c_s, c_i) = \left(\frac{\alpha_x\, \bar{\beta}_i\, h\, \bar{N}}{z} - \left(\tilde{f} + \alpha_{x1}\, \tilde{f}_0\right) h\, w\right)\left(\left(\tilde{f} + \alpha_{x1}\, \tilde{f}_0\right) R + \alpha_x\, \bar{N}\right)^{-1}. \quad (A2.12)$$

From (12) and the other equations, we get

$$c_i = H_1(z, c_s, c_i) \equiv \frac{\xi\, \hat{y}}{(1+\lambda_0)\,(P^{\xi_0} + \xi)\, \bar{\tau}_i},$$

$$c_s = H_2(z, c_s, c_i) \equiv \frac{\xi\, P\, \hat{y}}{(1+\lambda_0)\,(P^{\xi_0} + \xi)\, \bar{\tau}_i}. \quad (A2.13)$$

Suppose that we can solve (A13) with c_i and c_s as functions of z. Insert the obtained equations in $(A2.12)$:

$$\bar{k} = H(z). \quad (A2.14)$$

We thus obtain the following procedure as in the lemma: $\bar{k}$ by (A2.14) $\rightarrow$ r and w by (A2.2) $\rightarrow$ $r_\delta = r - \delta_k$ $\rightarrow p$ by (A2.3) $\rightarrow c_s$ and c_i by (A2.13) $\rightarrow \tau_s$ by (22) $\rightarrow \tau_i$ by (22) $\rightarrow \hat{y}$ by (10) $\rightarrow K_p$ by (A2.10) $\rightarrow K_s$ by (A2.7) $\rightarrow K = \bar{k}N \rightarrow K_i$ by (A2.6) $\rightarrow N_j$, $j = i, s, p \rightarrow F_j$ by the defined functions $\rightarrow U$ by (19) $\rightarrow Y_p$ by (5) $\rightarrow T_p = Y_p$. By (14) and this procedure, we have

$$\dot{\bar{k}} = \widetilde{\Psi}(z) \equiv s - \bar{k}. \quad (A2.15)$$

Taking derivatives of (A2.14) in time, we have:

$$\dot{\bar{k}} = \frac{\partial\, H}{\partial\, z}\, \dot{z}. \quad (A2.16)$$

From (A2.15) and (A2.16), we have:

$$\dot{z} = \widetilde{\Psi}\left(\frac{\partial H}{\partial z}\right)^{-1}. \quad (A2.17)$$

We thus proved the Lemma.

References

Auerbach, A.J. (1985) The Theory of Excess Burden and Optimal Taxation. In Handbook of Public Economics, vol. I, edited by Auerbach and Feldstein, M., pp. 69-127. Amsterdam: North-Holland.

Barro, R.J. (1990) Government Spending in a Simple Model of Endogenous Growth. Journal of Political Economy 98, S103-25.

Baldwin, R.E. and Krugman, P. (2004) Agglomeration, Integration and Tax Harmonization. European Economic Review 48, 1-23.

Burmeister, E. and Dobell, A.R. (1970) Mathematical Theories of Economic Growth. London: Collier Macmillan Publishers.

Diamond, P. and Mirrlees, J.A. (1971) Optimal Taxation and Public Production I: Production Efficiency. American Economic Review 61(1), 8–27.

Diamond, P. and Mirrlees, J.A. (1971a) Optimal Taxation and Public Production II: Tax Rules. American Economic Review 61(3), 261–78.

Gómez, M.A. (2008) Fiscal Policy, Congestion, and Endogenous Growth. Journal of Public Economic Theory 10, 595-622.

Ihori, T. and Yang, C.C. (2009) Interregional Tax Competition and Intraregional Political Competition: The Optimal Provision of Public Goods under Representative Democracy. Journal of Urban Economics 66(3), 210-17.

Lai, Y.B. (2019) Environmental Policy Competition and Heterogeneous Capital Endowments. Regional Science and Urban Economics 75(2019), 107-19.

Mas-Colell, A., Whinston, M.D., and Green, J.R. (1995) Microeconomic Theory. Oxford: Oxford University Press.

Mirrlees, J.A. (1991) An Exploration in the Theory of Optimal Income Taxation. Review of Economic Studies 38(2), 175-208.

Ramsey, F. (1927) A Contribution to the Theory of Taxation. Economic Theory 37, 47-61.

Slemrod, J. (1990) Optimal Taxation and Optimal Tax Systems. Journal of Economic Perspectives 4(Winter), 157-78.

Solow, R. (1956) A Contribution to the Theory of Growth. Quarterly Journal of Economics 70, 65-94.

Stiglitz, J.E. (1987) Pareto Efficient and Optimal Taxation and the New Welfare Economics. In Handbook of Public Economics 2, edited by Auerbach, A. and Feldstein, M., pp. 991-1042. Amsterdam: North-Holland.

Stiglitz, J.E. (2015) In Praise of Frank Ramsey's Contribution to the Theory of Taxation. The Economic Journal 125(583), Special Issue: 125th Anniversary Issue, 235-68.

Turnovsky, S.J. (2004) The Transitional Dynamics of Fiscal Policy: Long-Run Capital Accumulation and Growth. Journal of Money, Credit, and Banking 36, 883-910.

Uzawa, H. (1961) On a Two-Sector Model of Economic Growth. Review of Economic Studies 29, 47-70.

Wildasin, D.E. (1988) Nash Equilibria in Models of Fiscal Competition. Journal of Public Economics 35, 229-40.

Wilson, J.D. (1986) A Theory of Interregional Tax Competition. Journal of Urban Economics 19, 296-315.

Wilson, J.D. and Gordon, R. (2003) Expenditure Competition. Journal of Public Economic Theory 5(2), 499-417.

Zhang, W.B. (2005) Economic Growth Theory. London: Ashgate.

Zhang, W.B. (2008) International Trade Theory: Capital, Knowledge, Economic Structure, Money and Prices over Time and Space. Berlin: Springer.

Zhang, W.B. (2016) Public Debt and Economic Growth in Uzawa's Two-Sector Model with Public Goods. International Journal of Economic Sciences 5(4), 52-73.

Zhang, W.B. (2020) The General Economic Theory: An Integrative Approach. Switzerland: Springer International Publishing.

Zodrow, G.R. and Mieszkowski, P. (1986) Pigou, Tiebout, Property Taxation, and the Underprovision of Local Public Goods. Journal of Urban Economics 19, 356-70.